

# Study on $0^+$ states with open charm in unitarized heavy meson chiral approach

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We calculate the scattering amplitudes of Goldstone bosons off the pseudoscalar D-mesons in unitarized heavy meson chiral approach. The low energy constants appearing in  $\mathcal{O}(p^2)$  chiral Lagrangian are determined by fitting lattice simulations on  $S$ -wave scattering lengths.  $D_{s0}^*(2317)$  is obtained as a bound state in  $(S, I) = (1, 0)$   $DK$  channel. Possible bound states or resonance states in other channels are investigated as well. The quark mass dependence of the mass and binding energy of  $D_{s0}^*(2317)$  is also investigated, which indicates predominately  $DK$  molecular nature.

Keywords: Heavy Meson Scattering, Chiral Effective Approach, Chiral Extrapolation, Pole Analysis

## I. INTRODUCTION

In the last decade, the discovery of many narrow resonances with open charm open a new chapter in hadronic spectroscopy. Especially, the  $D_{s0}^*(2317)$  discovered by the BaBar Collaboration [1] and  $D_{s1}(2460)$  by the CLEO Collaboration [2] have inspired heated discussions both experimentally and theoretically. Moreover, the Belle Collaboration recently reported a broad  $0^+$  charmed meson with mass and width being  $m_{D_0^{*0}} = 2308 \pm 60 \text{ MeV}$  and  $\Gamma_{D_0^{*0}} = 276 \pm 99 \text{ MeV}$ , respectively [3]. Meanwhile, the FOCUS Collaboration reported a broad  $0^+$  charmed meson with mass and width being  $m_{D_0^{*0}} = 2407 \pm 56 \text{ MeV}$  and  $\Gamma_{D_0^{*0}} = 240 \pm 114 \text{ MeV}$ , respectively [4]. Although consistent with each other within the errors, it is still in dispute whether they are the same particle [5].

Possible interpretations of  $D_{s0}^*(2317)$  include normal  $c\bar{s}$  state [6], four-quark state [7], hadron molecular state [8], etc. To distinguish composite from elementary particles, different methods were proposed such as pole counting [9], scattering length and effective range [10]. As emphasized in a series of paper [11, 12], quark mass dependence of a state can also provide important information on its nature.

Effective field theories (EFTs) have been proven very successful in studying low energy hadron physics [13]. In light meson sector, chiral perturbation theory is an expansion in powers of external momenta and masses of Goldstone bosons [14, 15]. In high energy region or for large quark masses, chiral amplitudes violate unitarity severely. In addition, chiral expansion up to a given finite order does not contain resonance or bound state, which may modify the results of physical variables from perturbation theory significantly. Therefore, unitarized model was introduced to high energy region, in which lower lying scalar and vector resonances can be dynamically generated. Following the same spirit, heavy meson chiral perturbation theory (HMChPT) was proposed [16–18], and unitarization method was applied to some phenomenological analysis [19–23].

Lattice gauge theory is another powerful tool to study strong interactions. Lattice simulations are usually performed at unphysical quark masses, or equivalently at larger pion masses. Recently, lattice results for the charmed meson-light hadron scattering lengths are given at several chosen values of  $M_\pi/F_\pi$  [24]. These progresses can be used to make up the lack of experimental data on scattering processes. These lattice data can be used to determine the low energy constants in perturbative scattering amplitudes [25].

$D_{s0}^*(2317)$  as well as other possible charmed particles were investigated with the unitarized heavy meson chiral approach by studying the scattering lengths of charmed mesons and Goldstone bosons in Ref. [12, 23]. The quark mass dependence of the poles has an interesting behavior and provides a good way to understand the structure of the obtained poles. In their calculation, the large  $N_C$  approximation and the mass and width of  $D_{s0}^*(2317)$  as input are used to determine the low energy constants. In this paper, we use the similar approach to reinvestigate the charmed mesons scattering off light mesons. The difference between our treatment and theirs is that we do not apply the large  $N_C$  approximation. This is because in the real world,  $N_C$  is 3. Moreover, in the large  $N_C$  limit, the mass and width of the particle can be quite different from the real particle [26–28]. Therefore, three additional low energy constants (LECs)  $h_0$ ,  $h_2$  and  $h_4$  appear in our case. The parameter  $h_1$  can be determined by the mass difference between  $D$  mesons. The parameter  $h_0$  is obtained by the quark mass dependence of  $D$  and  $Ds$  mesons from the lattice data [29]. The other constants including  $h_3$  and  $h_5$  are determined by fitting the lattice data of the scattering lengths of  $D$  and light mesons [24]. To confirm the existence of  $D_{s0}^*(2317)$ , its mass and width are not used as input. All the states including  $D_{s0}^*(2317)$  will be obtained from pole analysis on the scattering amplitudes. As a comparison, the quark mass dependence of  $D_{s0}^*(2317)$  is also discussed.

The paper is organized as follows. In sect. II, the effective chiral Lagrangian up to next-to-leading order is briefly introduced. We calculate the unitary scattering amplitudes and determine the low energy constants by fitting lattice simulations on  $S$ -wave scattering lengths in sect. III. In sect. IV, we present the possible bound states or resonant poles in appropriate channels, and then investigate the quark mass dependence of  $D_{s0}^*(2317)$ . Finally, We make a brief summary in sect. V.

## II. THE EFFECTIVE LAGRANGIAN

The leading order chiral Lagrangian for describing the interaction between the Goldstone boson and the heavy pseudoscalar meson is [16–18]

$$\mathcal{L}^{(1)} = \mathcal{D}_\mu D \mathcal{D}^\mu D^\dagger - \overset{\circ}{M}_D^2 D D^\dagger \quad (1)$$

with  $D = (D^0, D^+, D_s^+)$ . The covariant derivative is

$$\begin{aligned} \mathcal{D}_\mu D^\dagger &= (\partial_\mu + \Gamma_\mu) D^\dagger, \\ \Gamma_\mu &= \frac{1}{2} (u^\dagger \partial_\mu u + u \partial_\mu u^\dagger), \end{aligned} \quad (2)$$

where

$$U = \exp\left(\frac{\sqrt{2}i\phi}{F}\right), \quad u^2 = U, \quad (3)$$

with  $\phi$  containing the Goldstone boson fields,

$$\phi(x) = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \quad (4)$$

$F$  is the Goldstone boson decay constant in the chiral limit, which we will identify with the pion decay constant,  $F = 92.4\text{MeV}$ .

The strong interaction part of NLO chiral Lagrangian reads

$$\begin{aligned} \mathcal{L}_{\text{str.}}^{(2)} &= D(-h_0\langle\chi_+\rangle - h_1\chi_+ + h_2\langle u_\mu u^\mu \rangle - h_3 u_\mu u^\mu) \bar{D} \\ &+ \mathcal{D}_\mu D(h_4\langle u^\mu u^\nu \rangle - h_5\{u^\mu, u^\nu\} - h_6[u^\mu, u^\nu]) \mathcal{D}_\nu \bar{D}, \end{aligned} \quad (5)$$

where  $\langle \rangle$  stands for the trace of the  $3 \times 3$  matrices, and

$$\begin{aligned} \chi_+ &= u^\dagger \chi u^\dagger + u \chi u, \\ u_\mu &= iu^\dagger \mathcal{D}_\mu U u^\dagger. \end{aligned} \quad (6)$$

with

$$\chi = 2B \cdot \text{diag}(m_u, m_d, m_s). \quad (7)$$

The  $h_1$  term is a little different from [23] in order that the term  $D\langle\chi_+\rangle\bar{D}$  will completely disappear in large  $N_C$  limit [19]. The corresponding coefficients  $C_1$  are also modified (see Tab. I). The term proportional to  $h_0$  leads to a singlet contribution to the  $D$ -meson masses which depends linearly on the light quark masses, and is the heavy meson analog of the pion-nucleon sigma term [30]. The  $h_1$  term will contribute to the  $SU(3)_V$ -violating mass splitting amongst  $D$  mesons. The masses of  $D$  and  $D_s$  mesons can be expressed as

$$\begin{aligned} M_D^2 &= \overset{\circ}{M}_D^2 + 4h_0 B(m_u + m_d + m_s) + 4h_1 B \hat{n}, \\ M_{D_s}^2 &= \overset{\circ}{M}_D^2 + 4h_0 B(m_u + m_d + m_s) + 4h_1 B m_s, \end{aligned} \quad (8)$$

from which we can determine

$$h_1 = \frac{M_{D_s}^2 - M_D^2}{4B(m_s - \hat{n})} = \frac{M_{D_s}^2 - M_D^2}{4(M_K^2 - M_\pi^2)} = 0.427, \quad (9)$$

$(S, I)$	Channel	$C_{LO}$	$C_1$	$C_{35}$	$C_0$	$C_{24}$
$(-1, 0)$	$D\bar{K} \rightarrow D\bar{K}$	-1	$3M_K^2$	-1	$-M_K^2$	-1
$(-1, 1)$	$D\bar{K} \rightarrow D\bar{K}$	1	$-3M_K^2$	1	$-M_K^2$	-1
$(0, \frac{1}{2})$	$D\pi \rightarrow D\pi$	-2	$-3M_\pi^2$	1	$-M_\pi^2$	-1
	$D\eta \rightarrow D\eta$	0	$-M_\pi^2$	$\frac{1}{3}$	$-M_\eta^2$	-1
	$D_s\bar{K} \rightarrow D_s\bar{K}$	-1	$-3M_K^2$	1	$-M_K^2$	-1
	$D\eta \rightarrow D\pi$	0	$-3M_\pi^2$	1	0	0
	$D_s\bar{K} \rightarrow D\pi$	$-\frac{\sqrt{6}}{2}$	$-\frac{3\sqrt{6}}{4}(M_K^2 + M_\pi^2)$	$\frac{\sqrt{6}}{2}$	0	0
$(0, \frac{3}{2})$	$D_s\bar{K} \rightarrow D\eta$	$-\frac{\sqrt{6}}{2}$	$\frac{\sqrt{6}}{4}(5M_K^2 - 3M_\pi^2)$	$-\frac{\sqrt{6}}{6}$	0	0
	$D\pi \rightarrow D\pi$	1	$-3M_\pi^2$	1	$-M_\pi^2$	-1
	$DK \rightarrow DK$	-2	$-6M_K^2$	2	$-M_K^2$	-1
	$D_s\eta \rightarrow D_s\eta$	0	$-2(2M_K^2 - M_\pi^2)$	$\frac{4}{3}$	$-M_\eta^2$	-1
	$D_s\eta \rightarrow DK$	$-\sqrt{3}$	$\frac{\sqrt{3}}{2}(3M_\pi^2 - 5M_K^2)$	$\frac{\sqrt{3}}{3}$	0	0
$(1, 1)$	$D_s\pi \rightarrow D_s\pi$	0	0	0	$-M_\pi^2$	-1
	$DK \rightarrow DK$	0	0	0	$-M_K^2$	-1
	$DK \rightarrow D_s\pi$	1	$-\frac{3}{2}(M_K^2 + M_\pi^2)$	1	0	0
$(2, \frac{1}{2})$	$D_sK \rightarrow D_sK$	1	$-3M_K^2$	1	$-M_K^2$	-1

TABLE I: The coefficients in the scattering amplitudes. Here,  $S$  ( $I$ ) denotes the total strangeness (isospin) of the two-meson system.

where  $\hat{m} = (m_u + m_d)/2$  and the mass relations of Goldstone bosons from leading order chiral expansion,

$$M_\pi^2 = 2B\hat{m}, \quad M_K^2 = B(\hat{m} + m_s), \quad M_\eta^2 = \frac{2}{3}B(\hat{m} + 2m_s), \quad (10)$$

are used. We can simply estimate the value of  $h_0$  to be 0.055 according to the slope of the extrapolation curve from lattice [29].

### III. SCATTERING AMPLITUDES AND UNITARIZATION

The perturbative chiral amplitudes up to NLO can be easily obtained. Besides the terms in Ref. [23] where large  $N_C$  suppressed ones are omitted, there are two additional terms (last two terms in Eq. (11)). Although suppressed in large  $N_C$  limit, the contributions from  $h_0$ ,  $h_2$  and  $h_4$  terms may not be negligible since we are working at  $N_C = 3$ . On the other hand, complete large  $N_C$  analysis in light meson sector shows that poles will move far away from their physical positions as increasing  $N_C$  [26–28]. Therefore, in this paper we include the complete tree level amplitude with definite strangeness and isospin up to  $\mathcal{O}(p^2)$ , which can be written as

$$\begin{aligned} T(s, t, u) &= T^{(1)}(s, t, u) + T^{(2)}(s, t, u) \\ &= \frac{C_{LO}}{4F^2}(s - u) + \frac{2C_1}{3F^2}h_1 + \frac{2C_{35}}{F^2}H_{35}(s, t, u) \\ &\quad + \frac{4C_0}{F^2}h_0 + \frac{2C_{24}}{F^2}H_{24}(s, t, u), \end{aligned} \quad (11)$$

where the subscripts denote the chiral dimension and the functions  $H_{35}$  and  $H_{24}$  are expressed as

$$\begin{aligned} H_{35}(s, t, u) &= h_3 p_2 \cdot p_4 + h_5(p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3), \\ H_{24}(s, t, u) &= 2h_2 p_2 \cdot p_4 + h_4(p_1 \cdot p_2 p_3 \cdot p_4 + p_1 \cdot p_4 p_2 \cdot p_3). \end{aligned} \quad (12)$$

Here, we adopt the same convention for the isospin decompositions as Ref. [23]. The coefficients in all the amplitudes are given in Tab. I. We also dropped the  $h_6$  term as in Ref. [23] since it is suppressed by one order due to the commutator structure. The tree level amplitudes can be projected to the  $S$ -wave by using

$$V_{ij}^{(S,I)}(s) = \frac{1}{2} \int_{-1}^1 d\cos\theta T_{ij}^{(S,I)}(s, t(s, \cos\theta), u(s, \cos\theta)), \quad (13)$$

where

$$u(s, \cos \theta) = m_1^2 + m_4^2 - \frac{1}{2s}[s + m_1^2 - m_2^2][s + m_4^2 - m_3^2] - \frac{1}{2s}\sqrt{\lambda(s, m_1^2, m_2^2)\lambda(s, m_3^2, m_4^2)}\cos \theta, \quad (14)$$

with

$$\lambda(s, m_i^2, m_j^2) = [s - (m_i + m_j)^2][s - (m_i - m_j)^2]. \quad (15)$$

In [31–33], a general method was proposed to construct scattering amplitudes satisfying unitarity, i.e.

$$T(s) = V(s)[1 - G(s) \cdot V(s)]^{-1}, \quad (16)$$

where  $V$  is a matrix whose elements are given by Eq. (13) and  $G$  is a diagonal matrix with the element being a two-meson integral

$$G_{ii}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2 - m_1^2 + i\epsilon} \frac{1}{(p_1 + p_2 - q)^2 - m_2^2 + i\epsilon}, \quad (17)$$

with  $m_1$  and  $m_2$  the masses of the particles appearing in the loop. The analytic expression of  $G_{ii}(s)$  can be expressed by [33]

$$G_{ii} = \frac{1}{16\pi^2} \left\{ a(\mu) + \log \frac{m_1^2}{\mu^2} + \frac{\Delta - s}{2s} \log \frac{m_1^2}{m_2^2} + \frac{\sigma}{2s} [\log(s - \Delta + \sigma) + \log(s + \Delta + \sigma) - \log(-s + \Delta + \sigma) - \log(-s - \Delta + \sigma)] \right\}, \quad (18)$$

where

$$\sigma = [-(s - (m_1 + m_2)^2)(s - (m_1 - m_2)^2)]^{1/2}, \quad \Delta = m_1^2 - m_2^2. \quad (19)$$

$a(\mu)$  is the subtraction constant with  $\mu$  the regularization scale. In the numerical calculation, we tried three possible values of  $a(m_D)$  estimated in Ref. [21]. Here, we did not use the the inverse amplitude method (IAM) to get the unitary scattering amplitude. For the light meson scattering, the divergence of one loop contributions from the leading order ( $\mathcal{O}(p^2)$ ) Lagrangian can be subtracted from the next leading order ( $\mathcal{O}(p^4)$ ) tree diagram resulting in the renormalized low energy constants. Since there are a lot of experimental and lattice data, the low energy constants for light mesons are well determined. However, for heavy meson, if we use the IAM to get the unitary scattering amplitude, the low energy constants at  $\mathcal{O}(p^3)$  are needed to cancel the divergence. The current data for heavy meson are not enough to determine all the constants at  $\mathcal{O}(p^2)$  and  $\mathcal{O}(p^3)$  very well. Therefore, in this paper, we apply the  $T$ -matrix formalism proposed in Refs. [31–33], where only one parameter, the subtraction constant  $a(\mu)$  was introduced. From Eq. (21), one can see that  $G(s_{thr})$  is up to  $\mathcal{O}(p^1)$ . If we include a linear dependence term of  $m_\pi^2$  in  $a(\mu)$ , as pointed out in Ref. [12], this higher order contribution would not change the general features of the results.

The  $S$ -wave scattering length is defined as

$$a_0 = -\frac{1}{8\pi(M_1 + M_2)} T_{thr}, \quad (20)$$

with  $M_1$  and  $M_2$  denoting the masses of the scattered heavy and light mesons, respectively.  $T_{thr}$  is the unitarized amplitude at threshold,  $s_{thr} = (M_1 + M_2)^2$ , which can be obtained from Eq.(16), with

$$V(s_{thr}) = \frac{1}{F^2} \left[ C_{LO} M_1 M_2 + \frac{2C_1}{3} h_1 + 2C_{35} (h_3 M_2^2 + 2h_5 M_1^2 M_2^2) + 4C_0 h_0 + 4C_{24} (h_2 M_2^2 + h_4 M_1^2 M_2^2) \right],$$

$$G(s_{thr}) = \frac{1}{16\pi^2} \left[ a(\mu) + \frac{1}{M_1 + M_2} \left( M_1 \ln \frac{M_1^2}{\mu^2} + M_2 \ln \frac{M_2^2}{\mu^2} \right) \right]. \quad (21)$$

There are no experimental data for the scattering of Goldstone bosons off  $D$ -mesons available. However, the low energy constants entering into NLO Lagrangian  $\mathcal{L}_{str}^{(2)}$  can be determined by fitting the recent lattice simulations on

	Fit I	Fit II	Fit III
$a(m_D)$	-0.373	-0.630	-0.864
$h_2$	$-0.216 \pm 0.022$	$-0.195 \pm 0.028$	$-0.127 \pm 0.025$
$h_3$	$0.393 \pm 0.180$	$0.510 \pm 0.320$	$-0.015 \pm 0.240$
$h_4$	$0.061 \pm 0.007$	$0.056 \pm 0.007$	$0.038 \pm 0.005$
$h_5$	$-0.001 \pm 0.020$	$0.032 \pm 0.014$	$0.172 \pm 0.060$
$\chi^2_{d.o.f}$	43.4/12 = 3.6	43.6/12 = 3.6	42.1/12 = 3.5

TABLE II: The fit results on LECs corresponding to different values of  $a(m_D)$  taken from Ref. [21].  $h_2$  and  $h_3$  are dimensionless, while  $h_4$  and  $h_5$  are in unit of  $\text{GeV}^{-2}$ .

(S, I)	Channel	LO	NLO	UChPT	CUCChPT	Lattice [24]
(-1, 0)	$D\bar{K} \rightarrow D\bar{K}$	0.36	$0.54^{+0.14}_{-0.15}$	$-1.24^{+0.41}_{-1.04}$		-0.23(4)
(-1, 1)	$D\bar{K} \rightarrow D\bar{K}$	-0.36	$-0.49^{+0.15}_{-0.14}$	$-0.21^{+0.03}_{-0.03}$		
(0, $\frac{1}{2}$ )	$D\pi \rightarrow D\pi$	0.24	$0.23^{+0.01}_{-0.01}$	$0.41^{+0.05}_{-0.04}$	$0.39^{+0.04}_{-0.04}$	
	$D\eta \rightarrow D\eta$	0	$-0.06^{+0.07}_{-0.07}$	$-0.05^{+0.06}_{-0.05}$	$-1.48^{+0.26}_{-0.44} + i0.04^{+0.20}_{-0.01}$	-0.16(4)
	$D_s\bar{K} \rightarrow D_s\bar{K}$	0.36	$0.23^{+0.15}_{-0.15}$	$0.58^{+0.78}_{-0.47}$	$-0.67^{+0.07}_{-0.44} + i0.10^{+0.38}_{-0.02}$	
(0, $\frac{3}{2}$ )	$D\pi \rightarrow D\pi$	-0.12	$-0.13^{+0.01}_{-0.01}$	$-0.10^{+0.01}_{-0.01}$		
(1, 0)	$DK \rightarrow DK$	0.72	$0.46^{+0.27}_{-0.27}$	$-1.99^{+1.22}_{-0.39}$	$-0.73^{+0.18}_{-0.55}$	0.00(1)
	$D_s\eta \rightarrow D_s\eta$	0	$-0.16^{+0.23}_{-0.23}$	$-0.11^{+0.20}_{-0.08}$	$-0.35^{+0.07}_{-0.09} + i0.05^{+0.11}_{-0.03}$	
(1, 1)	$D_s\pi \rightarrow D_s\pi$	0	$0.003^{+0.001}_{-0.002}$	$0.003^{+0.001}_{-0.002}$	$0.010^{+0.008}_{-0.002}$	
	$DK \rightarrow DK$	0	$0.02^{+0.02}_{-0.01}$	$0.03^{+0.01}_{-0.02}$	$-0.52^{+0.08}_{-0.02} + i0.22^{+0.31}_{-0.11}$	-0.31(2)
(2, $\frac{1}{2}$ )	$D_sK \rightarrow D_sK$	-0.36	$-0.51^{+0.15}_{-0.14}$	$-0.22^{+0.03}_{-0.02}$		

TABLE III: The S-wave scattering lengths from calculations at LO and NLO (units are fm). The results using unitarized amplitudes are also given in the two columns denoted by UChPT and CUCChPT, representing one-channel and coupled-channel unitarized chiral perturbation theory, respectively. LECs are taken from Fit III.

S-wave scattering lengths [24]. The lattice spacing is  $b = 0.12\text{fm}$ . The  $s$  quark mass is  $80\text{MeV}$ , which is consistent with its physical mass, and four ensembles are chosen with  $M_\pi = 0.1842, 0.2238, 0.3113, 0.3752$  in lattice unit, or equivalently  $M_\pi = 0.2925, 0.3554, 0.4943, 0.5958$  in unit of  $\text{GeV}$  (These lattice data were misused in [23]).

From Eq. (8), the pion mass dependence of  $D$  mesons up to  $\mathcal{O}(M_\pi^2)$  can be expressed as

$$\begin{aligned}
M_D(M_\pi) &= M_{D|phy} + \frac{2h_0 + h_1}{M_{D|phy}}(M_\pi^2 - M_{\pi|phy}^2), \\
M_{Ds}(M_\pi) &= M_{Ds|phy} + \frac{2h_0}{M_{Ds|phy}}(M_\pi^2 - M_{\pi|phy}^2),
\end{aligned} \tag{22}$$

and from Eq. (10) we can get

$$M_K(M_\pi) = \overset{\circ}{M}_K + \frac{M_\pi^2}{4\overset{\circ}{M}_K}, \quad M_\eta(M_\pi) = \overset{\circ}{M}_\eta + \frac{M_\pi^2}{6\overset{\circ}{M}_\eta}, \tag{23}$$

where  $\overset{\circ}{M}_K = 486\text{MeV}$  and  $\overset{\circ}{M}_\eta = 542\text{MeV}$  are the masses of kaon and  $\eta$  in the chiral limit, respectively. The physical masses for all mesons are taken from PDG [34], i.e.,  $M_{\pi|phy} = 138\text{ MeV}$ ,  $M_{K|phy} = 496\text{ MeV}$ ,  $M_{\eta|phy} = 548\text{ MeV}$ ,  $M_{D|phy} = 1867\text{ MeV}$ , and  $M_{Ds|phy} = 1968\text{ MeV}$ . Thus, with Eqs. (20)-(23), the scattering length  $a_0$  can be expressed as a function of  $M_\pi$ . The four unknown low energy constants  $h_2$ ,  $h_3$ ,  $h_4$  and  $h_5$  can be determined by fitting the lattice data of scattering lengths.

Although  $(S, I) = (1, 1)$  is in fact a couple channel case, we use single channel unitarization in our fit procedure for all of the four channels, since so far lattice data only exist without channel coupling. The obtained LECs in three cases are shown in Tab. II, corresponding to different values of  $a(m_D)$  which were estimated by comparing the dispersion relation method with the cut-off method [21].  $a(m_D)$  chosen to be  $-0.373$ ,  $-0.630$  and  $-0.864$  correspond to the resultant cut-off momentum  $q_{max}$  at  $0.6$ ,  $0.8$  and  $1.0\text{ GeV}$ , respectively. The obtained scattering length at physical pion mass for each channel are listed in Tab. III. In Fig. 1, we plot the fitted results of the scattering lengths

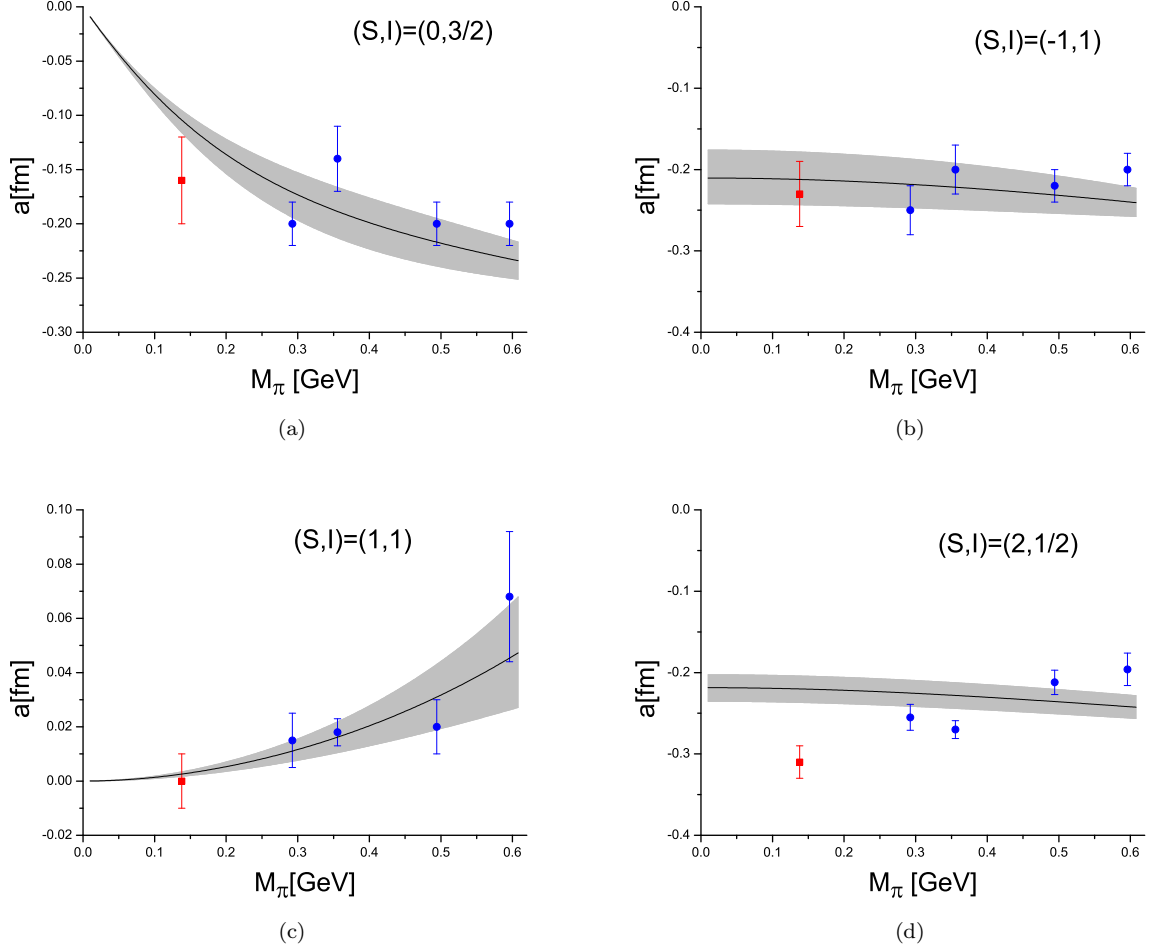


FIG. 1: The fitted results on scattering lengths. The squares at physical pion mass denote chiral extrapolation results given by [24], which are *not* included in our fit. The LECs are taken from fit III. Solid lines correspond to the central value of each parameter.

versus pion mass with  $a(m_D) = -0.864$  since all the three choices of  $a(m_D)$  give similar curves. One can see that lattice data can be fitted very well except the  $(S, I) = (2, 1/2)$  channel. The scattering lengths of  $(S, I) = (0, 3/2)$  and  $(S, I) = (1, 1)$  channels are sensitive to pion mass which can be understood from Tab. 1. In the chiral limit, both of them go to zero. The scattering lengths of  $(S, I) = (-1, 1)$  and  $(S, I) = (2, 1/2)$  channels change little with the increasing pion mass. The calculation with the heavy meson chiral perturbation theory is not completely consistent with the lattice data of  $(S, I) = (2, 1/2)$  channel which deserves further investigation. Without the large  $N_C$  suppression terms, the scattering length of  $(S, I) = (1, 1)$  channel remains zero with the increasing pion mass. In Ref. [23], with their notation, the coefficient  $C_1 = 2M_\pi^2$  makes the scattering lengths of  $(S, I) = (1, 1)$  channel be always negative and decrease with increasing pion mass. The inclusion of  $N_C$  suppression terms improves the fit of lattice data. Due to the inaccuracy of lattice data, we show the uncertainty of the scattering lengths in Fig. 1. The corresponding error bars of the low energy constants are also listed in Tab. II. For  $h_2$  and  $h_4$ , the errors are about 10% – 20% and their signs keep the same with all the three parameter sets. While for  $h_3$  and  $h_5$ , their signs could change with different parameter sets and the error bars are larger.

(S,I)	Channel	Thr	RS	Fit I	Fit II	Fit III
(-1,0)	$D\bar{K} \rightarrow D\bar{K}$	2363	I			$2340^{+20}_{-24}$
$(0, \frac{1}{2})$	$D\pi \rightarrow D\pi$	2005	II	$2315^{+30}_{-83} - i70^{+78}_{-70}$	$2194^{+117}_{-50}$	
	$D\eta \rightarrow D\eta$	2415	III	$2146^{+9}_{-8} - i124^{+14}_{-10}$	$2122^{+11}_{-9} - i93^{+13}_{-10}$	$2104^{+15}_{-13} - i75^{+19}_{-13}$
	$D_s\bar{K} \rightarrow D_s\bar{K}$	2464		$2478^{+28}_{-14} - i23^{+5}_{-5}$	$2434^{+9}_{-7} - i19^{+11}_{-7}$	$2376^{+15}_{-8} - i2^{+17}_{-1}$
(1,0)	$DK \rightarrow DK$	2363	I	$2356^{+6}_{-9}$	$2327^{+23}_{-19}$	$2295^{+40}_{-38}$
	$D_s\eta \rightarrow D_s\eta$	2516				
(1,1)	$D_s\pi \rightarrow D_s\pi$	2106	II	$2433^{+50}_{-31} - i26^{+6}_{-9}$	$2372^{+38}_{-25} - i39^{+2}_{-6}$	$2318^{+39}_{-28} - i37^{+2}_{-3}$
	$DK \rightarrow DK$	2363				

TABLE IV: Pole positions on  $\sqrt{s}$  plane in unit of MeV. Thr and RS denote channel threshold and Riemann Sheet, respectively.

#### IV. POLE ANALYSIS

Under the present convention, the relationship between  $S$  matrix and  $T$  matrix is given by

$$S_{ij} = \delta_{ij} - \frac{2i}{8\pi} \frac{\sqrt{k_i k_j}}{\sqrt{s}} T_{ij}(s), \quad (24)$$

with  $k_i$  the  $i$ -th channel momentum. In general, for a system with  $N$  open channels, we have in total  $2^N$  Riemann sheets, which can be enumerated as  $L(\sigma_1, \sigma_2, \dots, \sigma_N)$  where  $\sigma_i$  stands for the sign of  $\text{Im}k_i$  [35]. Taking 2-channel case for example, we enumerate the 4 sheets in the following way,

$$\begin{aligned} \text{sheet I : } & \text{Im}k_1 > 0, \text{Im}k_2 > 0, L(++) , \\ \text{sheet II : } & \text{Im}k_1 < 0, \text{Im}k_2 > 0, L(-+) , \\ \text{sheet III : } & \text{Im}k_1 < 0, \text{Im}k_2 < 0, L(--) , \\ \text{sheet IV : } & \text{Im}k_1 > 0, \text{Im}k_2 < 0, L(+-) . \end{aligned} \quad (25)$$

The analytic continuation of  $S$  matrix to different sheets can be obtained

$$S^{II} = \begin{pmatrix} \frac{1}{S_{11}} & \frac{iS_{12}}{S_{11}} \\ \frac{iS_{12}}{S_{11}} & \frac{\det S}{S_{11}} \end{pmatrix}, \quad S^{III} = \begin{pmatrix} \frac{S_{22}}{\det S} & \frac{-S_{12}}{\det S} \\ \frac{-S_{12}}{\det S} & \frac{S_{11}}{\det S} \end{pmatrix}, \quad S^{IV} = \begin{pmatrix} \frac{\det S}{S_{22}} & \frac{-iS_{12}}{S_{22}} \\ \frac{-iS_{12}}{S_{22}} & \frac{1}{S_{22}} \end{pmatrix}, \quad (26)$$

from which we can see that the poles on sheet-II and sheet-III correspond to zeroes of  $S_{11}(s)$  and  $\det S = S_{11}S_{22} - S_{12}S_{21}$ , respectively.

Corresponding to each set of parameters given by Tab. II, we list the pole positions found in appropriate channels in Tab. IV, from which one can see all the three parameter sets give similar results except for the  $(S, I) = (-1, 0)$  channel. The error bars of the mass and width come from the uncertainty of the low energy constants  $h_i$ .

In  $(S, I) = (-1, 0)$  channel, the pole structure is unstable. Pole positions are dependent on the strength of interactions, which is governed by LECs. In fit I, we find a ‘‘resonance’’, whose position is similar to [23]. However, it has no physical correspondence since a particle with mass below the lowest hadron-hadron threshold can not possess finite width by strong decay. In fit II, there is a virtual state, which is located on the real axis below  $D\bar{K}$  threshold on the second Riemann sheet. The bound state pole predicted by fit III is in agreement with [20]. Further experiments on this channel will determine which parameter set is more reasonable.

In  $(S, I) = (0, 1/2)$  channel, we perform 3-channel unitarization. For example, for the parameter set II, we find a broad second sheet pole at  $(2122^{+11}_{-9} - i93^{+13}_{-10})\text{MeV}$  and a narrow third sheet pole at  $(2434^{+9}_{-7} - i19^{+11}_{-7})\text{MeV}$ , respectively. Although still deviate from the experimental data [3, 4], our results are in agreement with [20, 21]. Ref. [22] gave some arguments to explain why resonances predicted theoretically have not been observed by experiment. In addition to production rate, finding a new state experimentally also depends on data measurements and analysis, which are affected by many factors, such as data statistics, the background, etc.

A bound state pole of  $D_{s0}^*(2317)$  in  $(S, I) = (1, 0)$  channel is obtained. In Ref. [12, 23], the bound state of  $D_{s0}^*(2317)$  is assumed and its mass is used as an input to determine the LECs. Here,  $D_{s0}^*(2317)$  is really obtained from the analysis of the poles. For example, for the parameter set II, the mass of  $D_{s0}^*(2317)$  in our analysis is  $m = 2327^{+23}_{-19}\text{MeV}$ , which is in agreement with [21].

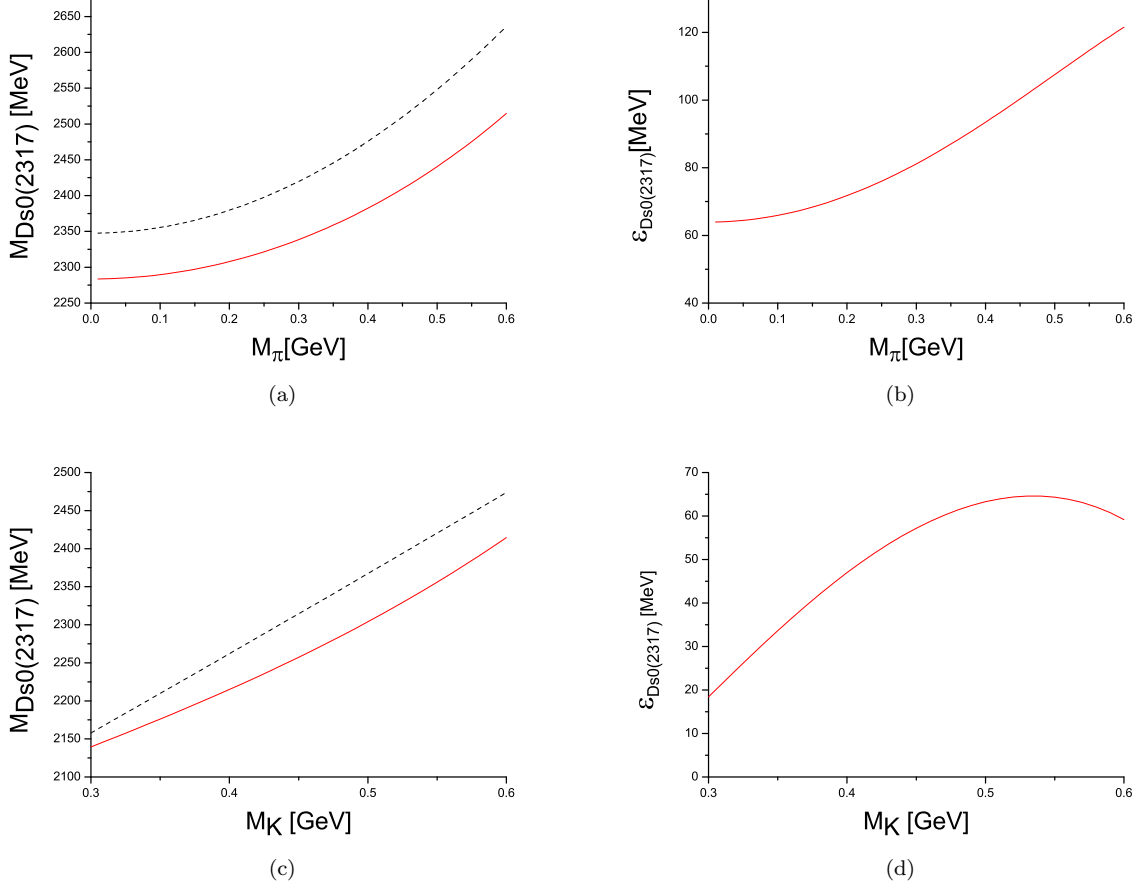


FIG. 2: a) Pion mass dependence of the mass of  $D_{s0}^*(2317)$ (solid) and  $DK$  threshold(dashed); b) Pion mass dependence of binding energy; c) Kaon mass dependence of the mass of  $D_{s0}^*(2317)$ (solid) and  $DK$  threshold(dashed); d) Kaon mass dependence of binding energy.

In  $(S, I) = (1, 1)$  channel, only  $N_C$  suppressed terms contribute to the elastic scattering amplitudes, as can be seen from Tab. I. The nearest resonance pole to the physical region is located on sheet II at  $(2372_{-25}^{+38} - i39_{-6}^{+2})\text{MeV}$ . The position of this state is different from that obtained in Ref. [21] where a resonance on sheet III with smaller mass and larger width exists.

Although the mass and width of the states are obtained, we did not get the information about the nature of the states. There are some methods to determine the structure of a particle, as mentioned in the introduction. In Ref. [12], the authors studied the structure by investigating the quark mass dependence of the states. This method provides a direct and clear picture for the composition of a particle. We now make the similar analysis as in Ref. [12]. We first fix  $s$  quark mass and vary the light quark masses. In Fig. 2a and 2b, we show the mass of  $D_{s0}^*(2317)$ , as well as the binding energy as a function of the pion mass. Both the mass and binding energy increase with the increasing pion mass. A pure  $c\bar{s}$  state has no constituent light quarks. Its light quark mass dependence only comes from sea quark contributions, which should be very weak as the case of  $Ds(1968)$  shown in lattice simulations [29]. The sensitive dependence of light quark mass shows that  $D_{s0}^*(2317)$  may probably be a  $DK$  molecular or tetraquark state where the constituent light quark exists.

Some general discussions on the importance of kaon mass dependence have been made in Ref. [12]. The mass of kaon-hadron molecular state is given by

$$M = M_K + M_h - \epsilon, \quad (27)$$

where  $M_h$  is the mass of the other hadron and  $\epsilon$  is the binding energy. The leading kaon mass dependence of such a bound state is linear, and the slop is unity.

To study the kaon mass dependence of  $D_{s0}^*(2317)$ , we fix the pion mass at its physical value, and express all results



in terms of  $M_K$ . From Eq. (8) and (10), we can obtain

$$\begin{aligned} M_D(M_K) &= M_{D|phy} + \frac{2h_0}{M_{D|phy}}(M_K^2 - M_{K|phy}^2), \\ M_{D_s}(M_K) &= M_{D_s|phy} + \frac{2(h_0 + h_1)}{M_{D_s|phy}}(M_K^2 - M_{K|phy}^2), \end{aligned} \quad (28)$$

and  $M_\eta(M_K) = \sqrt{\frac{4}{3}M_K^2 - \frac{1}{3}M_{\pi|phy}^2}$ .

Fig. 2c and 2d show the mass and binding energy of  $D_{s0}^*(2317)$  as a function of kaon mass. The  $K$ -meson mass dependence of  $D_{s0}^*(2317)$  is almost linear which is really in good agreement with the  $DK$  molecular expectation. These results are comparable with Refs. [12, 36].

## V. SUMMARY

In this paper, we calculate the complete scattering amplitudes of Goldstone bosons off the pseudoscalar D-mesons using unitarized heavy meson chiral approach. Two low energy constants  $h_0$  and  $h_1$  are determined by the mass splitting among  $D$  mesons and pion mass dependence of  $D$  and  $D_s$ . The other four LECs are determined by fitting lattice simulations on  $S$ -wave scattering lengths. The large  $N_C$  suppressed terms improve the fit. Three sets of parameters are obtained according to different choices of the subtraction constant  $a(m_D)$ . All the three sets of parameters give similar scattering lengths which are close to the lattice results.

For three parameter sets, the positions of the poles in each channel are close except in  $(S, I) = (-1, 0)$  channel. In this channel, the poles are sensitive to the parameter sets. Further experiments or lattice simulation can determine which parameter set is more reasonable. For other channels, we take the average values of the mass and width as the final results.  $D_{s0}^*(2317)$  is obtained as a bound state in  $(S, I) = (1, 0)$  channel, with the mass being  $m = 2326_{-22}^{+23}$  MeV. The strong pion mass dependence of its mass and binding energy disfavors conventional  $c\bar{s}$  content. The approximately linear kaon mass dependence reveals it is predominately a  $DK$  molecular state. In  $(S, I) = (0, 1/2)$  channel, a broad pole structure is found at  $(2124_{-10}^{+12} - i97_{-11}^{+15})$  MeV on the second Riemann sheet, and a narrow pole is at  $(2429_{-10}^{+17} - i15_{-4}^{+11})$  MeV on the third Riemann sheet. A resonance pole also exists on the second Riemann sheet with mass and half width  $2374_{-28}^{+42}$  MeV and  $34_{-6}^{+3}$  MeV in  $(S, I) = (1, 1)$  channel.

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